Finance researchers study the movement of stock prices.

One model is $P_{k+1} = P_k \left[1 + \mu t + Z \sigma \sqrt{t}\right]$ (Discrete-Time Model) where

- $P_k =$ stock price at time $k$ ($P_0$ is the initial stock price)
- $t =$ time period interval (in years or fraction thereof)
- $Z =$ standard normal random variable (normally distributed random variable with mean 0 and standard deviation 1)
- $\sigma =$ annualized standard deviation (“volatility”)
- $\nu =$ expected annual growth rate
- $\mu =$ instantaneous rate of return = $\nu + 0.50 \sigma^2$ (so $\nu = \mu - 0.50 \sigma^2$)

An alternate model, which is more elegant, is the following:

$$P_T = P_0 e^{(\mu - 0.50 \sigma^2)T + Z \sigma \sqrt{T}}$$ (Continuous-Time Model)

where $T =$ ending time (expressed in years from the beginning)

Black-Scholes Model for European Calls & Puts

Call Price = $SN(d_1) - X e^{-rT} N(d_2)$

where

- $S =$ current stock price
- $X =$ exercise price
- $T =$ duration of option (in years)
- $\sigma =$ annualized volatility (standard deviation)
- $N(y) =$ cumulative function for the standard normal distribution. That is, $N(y) =$ probability that a standard normal random variable is $\leq y$.

And

$$d_1 = \frac{\ln(S / X) + (r + \sigma^2 / 2)T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

The Put Price is $Put Price = X e^{-rT} N(-d_2) - SN(-d_1)$