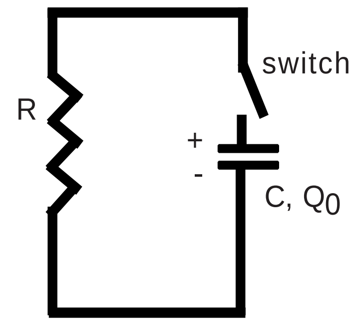
Name:

Resistor-Capacitor Circuits

- **Conceptual View: Discharging a Capacitor**
So far we have considered circuits with just capacitors or just resistors, now we consider a simple circuit with a switch, a capacitor, and a resistor, all in series. This is known as an RC circuit (R for resistor, C for capacitor). Imagine that the capacitor has been charged up to a charge Q_0, and the switch is then closed. (In all of the following feel free to use your analogies with water flow or traffic flow.)



1. Does current flow through the resistor after the switch is closed? Explain. If it does flow, show the direction that the "positive charge" (recall that it is really electrons that move, but "conventional current" has the protons moving) move through the circuit.

2. Does the current flow forever, or does it stop? Explain. Use your intuitions about the forces on charged particles.

3. Will the capacitor discharge faster or slower if I increase the resistance but keep all else the same? Explain.

4. Will the capacitor discharge faster or slower if I increase the capacitance and keep the initial charge the same? Explain.

5. Use Kirchhoff's loop rule (that is, the potential change around any loop is zero) to write the equation which describes the circuit. Your answer should have both a q_c (the charge on the capacitor) and an I_r (current through the resistor), as well as the paramters R and C.

6. $I_r \equiv dq_r/dt$ - it is the rate of flow of charge through the resistor. What is the relationship between dq_r/dt and dq_c/dt? Are they the same size? the same sign? Explain.

7. Put together your answers to find an equation for dq_c/dt in terms of q_c and the parameters of the problem. This is our differential equation for the charge on the capacitor.

8. Based on the differential equation, answer the following questions:

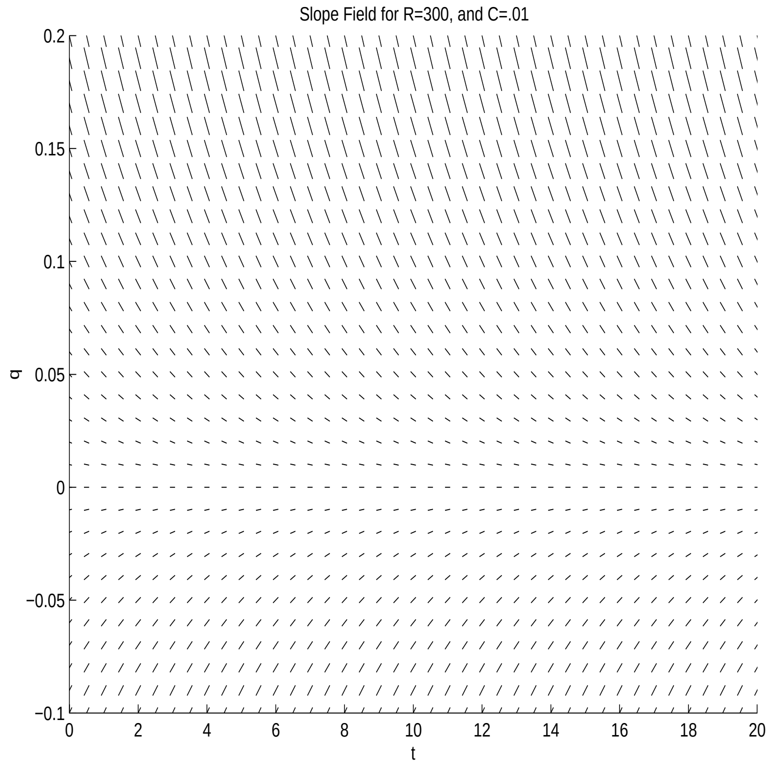
   What sign is the slope of the $q_c(t)$ curve?

   Does the slope of the curve $q_c(t)$ increase or decrease as the capacitor discharges?

   What is the initial charge on the capacitor?

   What is the charge on the capactior a long time after the switch is closed (i.e., what is the steady state)?

Below is the slope field for this differential equation.



9. Verify that this slope field corresponds to your differential equation.

10. Sketch the solution for an initial positive charge of .15 C.
11. Sketch the solution for an initial negative charge of $-.05$ C.
12. From the slope field, what is the steady state solution and does this agree with your intuitions?

13. Solve the differential equation analytically to find $q_c(t)$. You will need to use your initial value for q_c; take your "final" value to be q_c at time t.

14. In your equation, you should find that RC appear together. Define a new variable $\tau \equiv RC$ and rewrite the equation in terms of τ.

15. Show that τ has units of time. Hint: write R and C in terms of voltage, current, and charge.

16. At a time $t = \tau$, what is the ratio $q_c(t)/Q_0$?

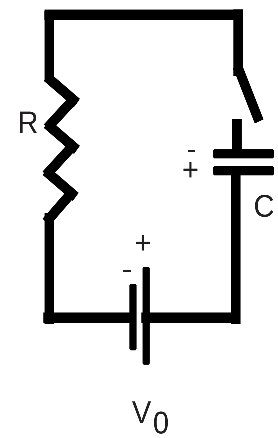
17. At a time $t = 2\tau$, what is the ratio $q_c(t)/Q_0$?

18. At a time $t = 3\tau$, what is the ratio $q_c(t)/Q_0$?

19. Use your last few answers to draw a slightly more accurate sketch of $q_c(t)$ - be sure to label the x-axis with τ, 2τ, 3τ.

τ is known as the time constant for this equation. It gives the time scale, that is, we know that significant changes occur over times comparable to τ.

20. Does the capactior discharge faster or slower as R and C increase? Does this agree with your original preciditons on the first page?



- Conceptual View: Charging a Capacitor Now consider the same circuit, but with a battery. Such a circuit would be used to charge up the capacitor. The capacitor is originally uncharged.

1. Use Kirchhoff's loop rule to write the equation which describes the circuit. Your answer should have both a q_c and an I_r and parameters.

2. What is the relationship between I_r and dq_r/dt? What is the relationship between dq_c/dt and dq_r/dt? Are they the same sign? Same magnitude?

3. Use your answers in the previous questions in order to give an equation for dq_c/dt in terms of q_c and the parameters of the problem. This is the differential equation for the problem.

4. Based on the differential equation, answer the following questions:

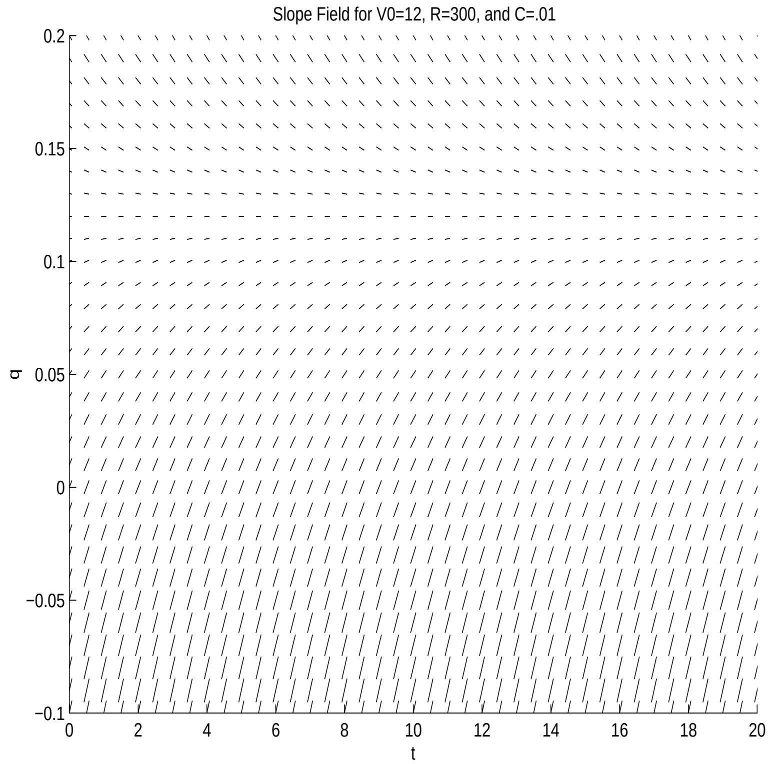
   What sign is the slope of the $q_c(t)$ curve?

   Does the slope of the curve $q_c(t)$ increase or decrease as the capacitor discharges?

   What is the initial charge on the capacitor?

   What is the charge on the capactior a long time after the switch is closed (i.e., what is the steady state)?

Below is the slope field for this differential equation.



5. Verify that this slope field corresponds to your differential equation.

6. Sketch the solution for an initial positive charge of .15 C.
7. Sketch the solution for an initial negative charge of $-.05$ C.
8. From the slope field, what is the steady state solution and does this agree with your intuitions?

9. Solve the differential equation to find $q_c(t)$, being sure to use the initial conditions.

10. What is the time constant in this problem?

Taking measurements In this experiment we will verify the prediction that we have made concerning the voltage across a capacitor as a function of time.

1. **Predictions**

   (a) Given that your resistor has $R = 1M\Omega$, and the capacitor has $C = 4.32\mu$F, calculate the time constant for this circuit. (Check your capcitor, it may be a bit different.)

   (b) In terms of battery voltage $V_0 = 4$ V, R and C, what is the maximum voltage you expect to see across the capacitor? What is the maximum charge?

   (c) How long will it take to charge up to 95% of that voltage?

   (d) Given your previous answer, sketch how you expect the voltage across the capacitor to change in time as it is being charged up.

   (e) Write an eqaution of $V_c(t)$ using the numerical values in this experiment.

2. **Setting up**

   (a) Turn on the Mac and open `Electricity`.

   (b) Put one graph on the screen to plot voltage. Fix the time scale and the voltage scales based on your answers to the previous part. (The battery voltage will be less than 5 Volts.)

   (c) Fix the settings under the `Collect` Menu:

      i. `All Graphs Live` (the graphs will be plotted as you take data

      ii. `Display Inputs` (the numerical value of the voltage) will be displayed on the bottom of the screen).

      iii. `Select Inputs`: Port1 only

      iv. `Data Rate`: 20 points per second

      v. `Averaging`: 15 point averaging for Port1

      vi. `Triggering`: Port1 > 0

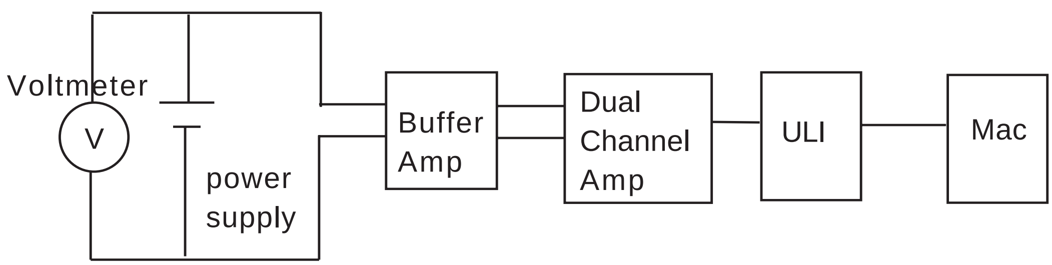
(d) Calibrate the voltages:

i. To calibrate the voltage you will need to measure the voltage directly across the power supply as shown. You will also need to hook up the voltmeter across the power supply to give the calibration voltage. Here are the details of the setup:

A. The buffer amplifier is the small homemade metal box with a switch. The connections on the "in" side go to the battery, the connections on the "out" side go to the dual channel amplifier. Turn on this amp before turing on the power supply.

B. selected (i.e, pushed in). For the voltmeter, one banana plug is placed in the "com" position, the other is placed in the "V/Ω" position. Turn the meter on.

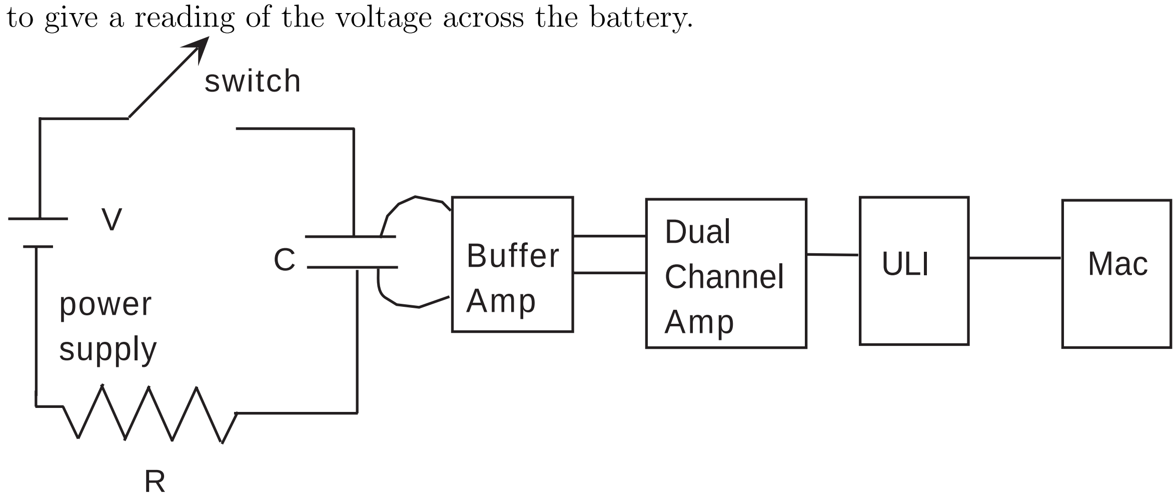
C. The Dual Channel amplifier is marked with its name. Be sure to use the "probe" pin connected to the banana plugs, not the ones connected to small boxes label "current".



ii. Under `Collect` choose `Calibrate`, then `Calibrate Now`. You need to take readings at two voltages; choose about 1V and 5V. Follow the instructions on the screen.

iii. Check that the calibration is correct by comparing the voltmeter and computer readings for several values between 1 and 5 volts.

iv. Hook up the circuit as shown. You may also keep the voltmeter attached to give a reading of the voltage across the battery.



v. Discharge the capacitor by running a short wire across the capacitor for a second or so. Verify that the voltage across the capacitor is zero (or nearly so) by looking at the computer reading.

3. **Taking Data**

   (a) Set the voltage between 4 and 4.5 Volts; keep it there for the remainder of the session.

   (b) Press start on the computer, wait about ten seconds to allow the ULI to turn on, then quickly and firmly close the switch.

   (c) Now we need to fit the data.

      i. Choose `Analyze Data A` from the `Analyze` menu. If a region of your data is bad (i.e., you started taking data before the switch was shut), be sure to highlight the good portion of your data before continuing.

      ii. If only some of your data is "good", select that section of the data.

      iii. Choose `Fit...` from the same menu.

      iv. On the left hand side of the `Fit` box, choose the functional form that you expect to see.

      v. Then choose `Try Fit`. Once the results are acceptable, choose `Maintain Fit`.

      vi. Print the plot if possible, otherwise, write down the fit here:

      vii. What is the value of τ from the fit?

      viii. What is the maximum voltage from the plot itself?

      ix. Are these in reasonable agreement with expectations?

   (d) Repeat your experiment, putting capacitors in series and parallel, or resistors in series and parallel. Be sure to predict the new time constant before you measure it.

Configuration	R value	C value	Predicted τ	Measured τ

   (e) **Please be sure that the buffer amplifier is turned off when you go (along with everything else!)**