Name:

Balance Point

In the following set of exercises we will learn how to find the balance point of an object. The balance point is the place where you can place your finger and be able to support the entire object. We will see in the next class that this point is also essential for understanding motion of the object.

1. Finding the Balance Point Experimentally

   Obtain a meter stick, two weight hangers and a set of weights. Place the weight hangers at the specified locations (measured from the center of the meter stick) and find the mass m_1 needed to make the center of the stick (here taken to be 0 cm) the balance point of the system. Note that the hangers themselves have a mass of about 20 kg; this must be included in m_1 and m_2. Finally, the smallest object we have has a mass of 10gm, so you will not be able to get the masses correct to better than 10 gm.

x_1	m_1	x_2	m_2
-20 cm		20 cm	70 gm
-20 cm		40 cm	20 gm
-20 cm		15 cm	40 gm

2. Verifying the formula for the balance point

   The book gives the following definition for finding the balance point:

$$x_{\text{balance point}} \equiv \frac{1}{M_{\text{total}}} \sum_{i=1}^{N} m_i x_i$$

   where N is the total number of objects that we are considering.

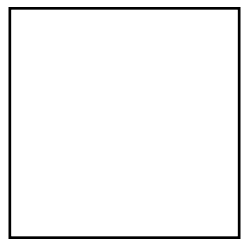
   In the following space verify that this formula gives the center of mass at the center of meter stick for each of the three situations given above. (Ignore the mass of the meter stick itself in this calculation.)

3. No-calculation balance points

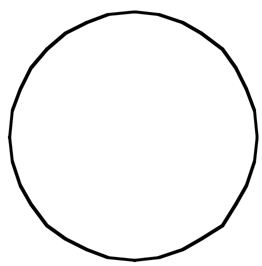
   In following three cases, imagine that the object is a thin sheet of metal of the specified shape and uniform density, and that you are trying to balance it with your finger while the object is horizontal.

   Mark the balance point of a meter stick with no extra hanging weights.

   Mark the balance point of a square book.



   Mark the balance point of a circle.



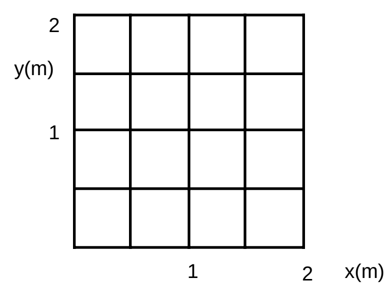
   How did you determine these balance points?

4. Calculating Balance points in two dimensions

   Now imagine that you have a square plate on which you place three objects with the following locations and masses:

x	y	m
0 m	0 m	3 kg
1 m	2 m	4 kg
2 m	1 m	8 kg

   Sketch the location of the objects on the coordinate system of the plate and use your intuition to guess the location of the balance point. Mark your guess on the sketch with a ”g”. (Ignore the mass of the plate itself in this calculation.)



   Using the above formula for x balance point and

$$y_{\text{balance point}} \equiv \frac{1}{M_{\text{total}}} \sum_{i=1}^{N} m_i y_i$$

   calculate the balance point and mark that on your sketch with an ’x’.

5. Calculating the Balance point in pieces

   To illustrate a useful technique we will calculate the balance point in a different way. First calculate the balance point of the first two objects only in the last section:

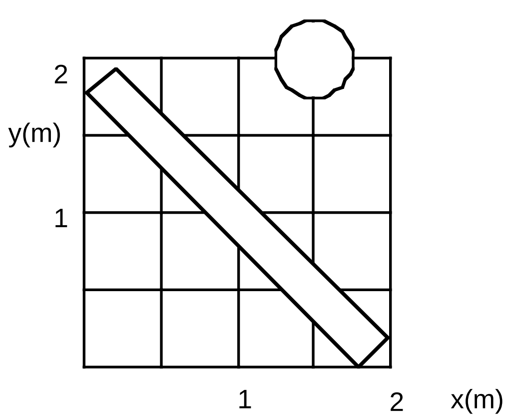
   Now, consider those first two objects to be one object with a mass of 7 kg located at their balance point. Now calculate the balance point for the "7 kg object" and the 8 kg object.

   You should find that the balance point calculated in this section and the last are the same. If not, go back and check your calculations.

   For those of you that like proofs, can you show that these balance points must be the same?

6. Calculating Balance points for complex objects

   Use the ideas developed in this sheet to calculate the balance point for the following situation. Be sure to indicate what you are doing and why. The mass of the rectangle is 5kg, the mass of the circle is 3kg.



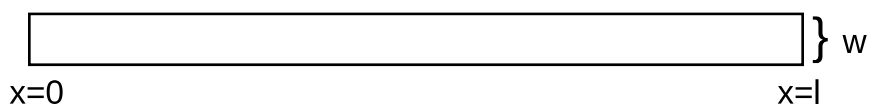
7. Calculating the Balance point for even more complex objects

Imagine that you have a rectangular sheet of length $l = 1$ m and width $w = 1/7$ m constructed so that the density is

$$\text{density} = 3x^2 = \text{mass/area}$$

Where x is the distance along the sheet, with $x = 0$ at the left side and 3 is in units of kg/m^4. With this density, the sheet is much heavier on the right end than on the left. Sketch the density *vs.* x below. Calculate the values at each end of the bar.

Mark with a "g" where you would guess the balance point is located.



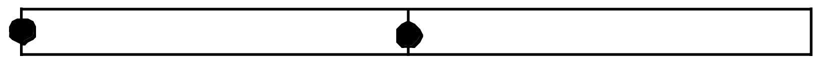
What is the value of $y_{\text{balance point}}$? Explain.

Calculating $x_{\text{balance point}}$ is more difficult and will take several steps. Why do none of our other methods that we have used so far work here?

In this case we will need to **approximate** the total mass ($\sum m_i$) and first moment of the mass (defined as $\sum m_i x_i$, the numerator in the center of mass formula) by breaking the sheet into pieces and approximating the density of each piece to be constant. (This is consistent with what we did in the last exercise: calculate the center of mass in pieces.) Do you expect this approximation to be accurate for two pieces? four pieces? one thousand pieces? Explain.

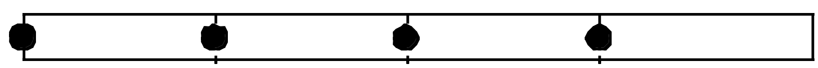
(a) To calculate $x_{\text{balance point}}$, first imagine that the mass of the sheet is located at only two points as illustrated below.

Fill out the chart below, keeping numbers as fractions, keep the powers explicit (e.g. you should have terms like $(1/4)^2$, not $1^2/8$). The reason for this is that we are looking for a general pattern which will not be obvious otherwise.



	piece 1	piece 2
Δx size of piece		
x location of dot		
density at the dot		
area of the pieces		
mass assuming constant density		
mx first moment of the mass		

(b) Do the same thing as in the previous question, considering that all the mass is located at four evenly spaced points along the sheet.



	piece 1	piece 2	piece 3	piece 4
Δx size of piece				
x location of dot				
density at the dot				
area of the pieces				
mass assuming constant density				
mx first moment of the mass				

(c) We have calculated the terms for the total mass $\sum m_i$ and first moment $\sum m_i x_i$. Rewrite the terms in this sum using the density instead of the mass.

(d) Based on your answers to the last two questions, come up with a general form for the terms of the i'th piece if you had n points.

	piece i $(0 \le i \le n)$
Δx	
x	
density	
area	
mass	
mx	

(e) Next you will use Matlab to calculate $x_{\text{balance point}}$ for several different values of n. Note that you can either take the points to be at the left edge $(0 \le i \le (n-1))$ or at the right edge $(1 \le i \le n)$. Don't forget to use the up arrow key to re-run the code for different values of n. Also, recall that `sum(f(2:N+1))` sums the second through `N+1` elements in the vector `f`.

Write out your matlab code below:

Write down the values obtained below:

n	total mass (left)	total mass (right)	mx (left)	mx (right)
2				
5				
10				
100				
1000				

(f) Why do the values of total mass and balance point change as you change the number of points? Why do they change less as the number of points gets larger?

8. Balance point of the bird

   An unusual bird will be passed around the class. Where do you think the balance point of the bird is? Hint: what other object behaves like the bird? Where is the center of mass of that object in relation to its support?

   Why does the bird behave as it does?

9. Balance point of the bottle

   Sketch the wine bottle and holder below. Where is the balance point? You can estimate the balance point from geometry and from looking at the wooden stand. Do those values agree?

10. Estimate of balance point

    Below is the sketch of another bird made out of a piece of wood of uniform density. Which of the numbered points is most likely to be the balance point? Hint: first elminate those that are obviously incorrect. For the rest, draw axes through the points, compare mass distribution above and below and left and right.