Physics problems for life scientists
Pushing on the Environment (aka collisions for biologists)

Overview of problem set: Animals move around by pushing on the environment (e.g., the ground, the air or the water). In these problems you will investigate quantitative details of this mechanism in several situations.

Possibly useful mathematics formulas:

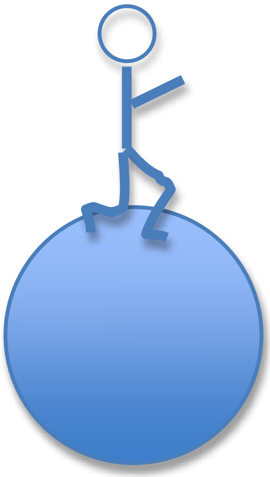
$$\text{Volume}_{\text{sphere}} = \frac{4}{3}\pi r^3 \quad \text{Volume}_{\text{cylinder}} = \pi r^2 h \quad \text{Area}_{\text{circle}} = \pi r^2$$

General Note to instructors: You will likely want to change the amount of scaffolding (step by step leading through the problem) that you give the students, either increasing or decreasing as is consistent with your student needs and your goals. Long problems can be shortened by doing some calculations for the students (e.g. geometry calculations may not be the most important for them to do but take time and effort).

Problems at a glance

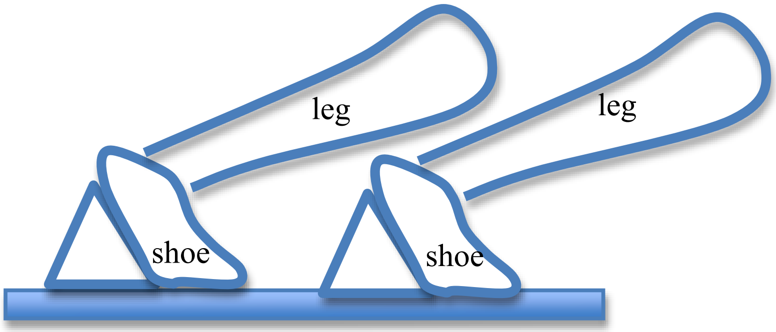
Biological system	**Physics Concepts**	**Math**	**Difficulty**	**Extensions**
Walking people	Conservation of momentum; impulse momentum; static friction	Algebra	Introductory	Conceptual
Hovering Hummingbird	Conservation of momentum; impulse momentum	Algebra and geometry	Difficult	Epistemological; extensions to hawks and scaling issues
Jetting jellyfish	Conservation of momentum; impulse momentum; buoyancy; drag	Algebra and geometry	Difficult	Epistemological; Does this work in air; why must intake be different; computational solution

1. **Walking people.** Big question: If we push leftward on the earth as we walk rightward, why doesn't the earth start moving?
    a. Biology Overview: Walking is an everyday event, so we don't think you need any overview here!
    b. Data: See below.



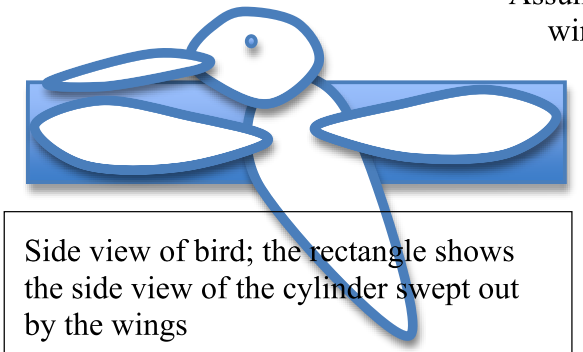
    c. Physics overview: Newton's third law says if you push on the earth, the earth pushes back on you with an equal and opposite force. This force from the earth what propels you forward.
    d. Question: The Little Prince (in the book by Antoine de St. Exupery) lives on a very small planet. Let's estimate that the planet has 100 times the mass as the prince, and assume the prince has a mass of 35 kg.
        i. Draw an interaction diagram of the Prince as he is walking. What is he touching? What forces are acting on him? Which force(s) are important for walking? If you push leftward on the earth with 10N, what force does the earth exert on you and how do you know (what physics principle is involved?)
        ii. If the little prince and his planet are initially at rest, and then he begins walking at a speed of .5 m/s, how fast is the planet moving? (Instructors: if you haven't covered rotations, you can model this as conservation of linear momentum.)
        iii. If the interaction time between him and the earth (the time over which he pushes) is .25 seconds, how great is the horizontal force that he exerts on the earth? Would he notice the motion of his planet?
        iv. Now imagine the little prince is vacationing on earth. If the little prince and the earth are initially at rest, and then he begins walking at a speed of .5 m/s, how fast is the earth moving? If the interaction time between him and the earth (the time over which he pushes) is .25 seconds, how great is the horizontal force that he exerts on the earth? Would he notice the motion of the earth? The earth has a mass of 6×10^{24} kg.

e. Extensions:
   i. (Conceptual) Why are runners' starting blocks designed as they are? That is, how do they help runners to achieve a large acceleration?

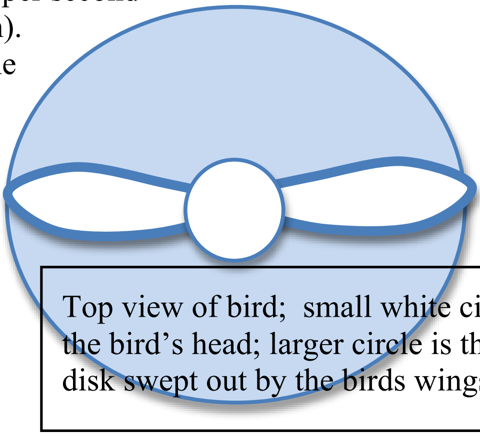


   ii. (Conceptual) Why is it hard to walk on smooth ice?
   iii. (Conceptual) Why do running shoes have rubber soles? (Think about running in fancy dress shoes.)
   iv. (Conceptual) Vehicles also move and steer by having the tires push on the road. Hydroplaning is when water builds up between your tires and the road. Why is this dangerous? Why is it dangerous for the tires to leave the road? (If you go quickly over the top of a hill you can actually leave the road.)

2. **Hovering Hummingbird.** Big questions: A hummingbird can hover in one place, but no other bird hovers in the same way. Can we quantitatively explain hovering using simple physics? How can hummingbirds change their hovering behavior as conditions (e.g. air density) change? What are the challenges to a bird 100 times larger hovering?
    a. Biology overview: When they hummingbirds hover, their bodies are nearly vertical, and their wings flap at an astounding 50 beats per second. Their wings move in half-circles parallel to the ground. In both the forward and backward wing movement, the upper edge of the wing leads the motion; no other bird wings work this way. (Check the internet for videos of hummingbirds to get a better sense of their motion; at this writing PBS and National Geographic have great videos.) You can nicely imitate a hummingbird while standing and moving your arms back and forth in horizontal arcs; unlike the hummingbird, you will not be able to move your arms 180 degrees (half a circle).
    b. Data: Assume our hummingbird weighs 3 gm, has a wing disk diameter of 10 cm, and beats their wings 45 times per second (one time includes back and forth motion). Assume the wing is angled so that the cylinder of air that it sweeps out is 0.5 cm tall, and that each wing makes a full 180° arc. (See D. L. Altshuler and R. Dudely, "Kinematics of hovering hummingbird flight along simulated and natural elevation gradients", Journal of Environmental Biology 206, 3139-3147, 2003.)



Side view of bird; the rectangle shows the side view of the cylinder swept out by the wings



Top view of bird; small white circle is the bird's head; larger circle is the wing disk swept out by the birds wings

    c. Physics overview: By Newton's third law, if the bird pushes down on the air, the air pushes up on the bird. Impulse momentum theorem: I can estimate the force on the air if I know the change in momentum of the air and the time period over which the momentum changed.
    d. Question: The hummingbird manages to hover (stay in place) because it pushes down on the air, and the air pushes back upward on the bird, and this upward force can be large enough to balance the weight of the bird. In this problem you will verify that this explanation is in reasonable agreement with the data given above. We begin with Newton's third law

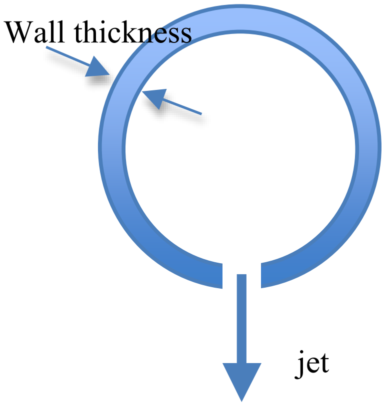
$$\vec{F}_{\text{of air on bird}} = -\vec{F}_{\text{of bird on air}}$$

Your job in this problem will be to evaluate the right hand side of this equation and use the above equality to see if the resulting force on the bird is large enough to balance its weight.

   i. Write down the impulse momentum theorem. The object being accelerated is the air next to the bird's wings. If you want to find the force of the bird on the air, what other quantities do you need to estimate? Outline how to estimate them from the data given.
   ii. What mass of air does the bird sweep every second? The volume of air is the volume of a cylinder. (Ignore the size of the body!)
   iii. If we assume the change in velocity of the air is equal to the velocity of the bird wing at the middle of the wing, what is the change in velocity of the air? (Hint: velocity = distance traveled over time; what distance does the middle of the wing travel in one back and forth motion? How long does that back and forth motion take?)
   iv. Use the impulse momentum theorem to find the average force exerted by the bird on the air. (Hint: since we know the change in velocity of the air – we assume it starts at rest – what is the change in momentum for the air in one back and forth motion? How long does that take?)
   v. Given your calculations and Newton's third law, is the net force on the bird zero? If the net force is not zero, what could the bird realistically do to make them equal?
   vi. This has been a long problem. What were the key ideas that you used?

b. Extensions:
   i. (Epistemological) This is an estimation problem – is it worth solving if it is only an estimation?
   ii. (Epistemological) What are the approximations in this model? Which approximations seem most inaccurate and why? Why did we ignore the buoyancy?
   iii. (Estimate) Estimate the power needed by the hummingbird to hover.
   iv. (Quantitative. Scaling) Few others birds hover. Kestrels hover, but in a very different way. In this problem we will consider an imaginary hovering hawk. Assume a mass of 21 oz= 600 g and a wing span of 3ft = 1m. Go back over your calculations for the hummingbird and substitute in values for the hawk. What would have to be the beat frequency for this hawk to hover? What power would be required? Can you give a quick scaling argument about why hovering can only be done by very small animals (insects hover, too).

3. **Jetting jellyfish.** Big question: What velocities can be achieved through biological jet propulsion (with and without considering drag)? Why is the water intake time longer than the water ejection time? What would be the challenges to an imaginary "jellybird" that gets around in the same manner, but in air?
    a. Biology overview: Jellyfish, squids, octopods, cuttlefish, scallops and Nautilus move around by taking in water and squirting it back out. We model the organism as a sphere with a small hole through which the water is expelled; a cross section is shown below. The water is ejected through the opening, resulting in a thrusting force in the opposite direction.



    b. Data from Mark Denny's "Air and Water" (Princeton University Press, 1993, pp 45-47). Assume a spherical jellyfish with r=10 cm, wall density = 1080 kg/m^3, thickness of wall = .7 cm, diameter of opening = 1.8 cm; time to eject water jet = 0.5 s; time to draw in water = 2 s

    c. Physics overview: If the jellyfish expels the water by pushing on it rightward, the water exerts an equal and opposite force leftward on the jellyfish. Impulse momentum theorem: I can estimate the force on a water if I know the change in momentum of the water and the time period over which the momentum changed.

    d. Question: The jellyfish accelerates because it pushes the water rightward and the water pushes back leftward. We begin with Newton's third law

$$\vec{F}_{\text{of water on jellyfish}} = -\vec{F}_{\text{of jellyfish on water}}$$

    Your job in this problem will be to evaluate the right hand side of this equation and use the above equality to calculate the acceleration of the jellyfish.
        i. Write down the impulse momentum theorem. The object being accelerated is the water in the organism. If you want to find the force of the fish on the water, what other quantities do you need to estimate? Outline how to estimate them from the data given.
        ii. What mass of water is held inside the fish?
        iii. Assuming a constant speed, how fast must the water move out of the fish in order for it to empty in the time specified? This is

a geometry question. Hint: what is volume flow rate through a pipe?

        iv. Use the impulse momentum theorem to find the average force exerted by the fish on the water.
        v. What is the resulting acceleration of the jellyfish if no other forces are acting? What is the final velocity after ½ s of acceleration?
        vi. To make this calculation more realistic, what other forces must be included? How are these forces going to change our results?

e. Extensions
    i. (Quantitative) When the jellyfish is filling up with water, what is the resulting acceleration (magnitude and direction), given the data above? Why must it fill more slowly than it ejects? What other mechanisms might give this same benefit?
    ii. (Epistemology) This is an estimation problem – is it worth solving if it is only an estimation?
    iii. (Epistemology) What are the approximations in this model? Which approximations seem most inaccurate and why?
    iv. (Conceptual) What challenges would there be to a bird trying to get around in this same manner? How might they overcome those challenges?
    v. (Computational. Advanced.) Model the whole motion, including drag force. Because the drag force changes with velocity, the force is not constant and the acceleration is not constant. One way to handle this (if you don't know differential equations) is to do a numerical approximation. Take tiny time steps (during each step acceleration is nearly constant) and use the constant acceleration equations to estimate velocity at the next time step. This is known as Euler's method. (Excel will do this for you nicely.)

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