Physics problems for life scientists
Forces in Fluids

Before these problems are assigned:

Possibly useful mathematics formulas:

$$\text{Volume}_{\text{sphere}} = \frac{4}{3}\pi r^3 \quad \text{Volume}_{\text{cylinder}} = \pi r^2 h \quad \text{Area}_{\text{circle}} = \pi r^2$$

Problems at a glance:

Biology	Physics	Math	Difficulty	Extensions
Falling Cats	pressure drag; Newton's second law	Algebra	Introductory	Falling mice and horses (scaling issues); Hitting the ground (impulse momentum)
Hydrodensiometry Percent body fat	Buoyancy, Newton's second law	Algebra	Introductory	Examining assumptions; affect of air in lungs; Siri formula
Centrifuge	Centripetal acc; buoyancy; viscous drag; Newton's second law; hydrostatic pressure			
Fish Bladders	Buoyancy, Newton's second law			Terminal velocity

1. **Falling cats.** Big question: Why do cats who fall more than seven stories have less injuries (on average) than cats who fall less than seven stories?
    a. Biology overview: Falling cats have two behavioral adaptations that help them survive. First, using their tail they are able to right themselves so they fall feet first. Second, once they reach terminal velocity (which means that acceleration is zero) they relax and adopt a “spread eagle” position, thereby increasing their effective area and reducing their terminal velocity.
    b. Data: We’ll let you come up with the data needed here (mass and area of an average cat, mouse and horse) based on your own experience. Resources: Diamond, J. 1989. How cats survive falls from New York skyscrapers. *Natural History* 98:20-26. Data in this paper are based on veterinary records of accidental falls - not deliberate defenestration of felines.
    c. Physics overview: There are two forces acting on a falling cat: weight downward and pressure drag upward. The pressure drag increases as the velocity increases. Terminal velocity is reached when acceleration is zero, because if acceleration is zero, velocity remains constant.
    d. Question: What is the terminal velocity for a cat, mouse and a horse? Estimate the terminal velocity for a cat both in the spread eagle position and the “standing” position (with legs straight down). Estimate any data you need based on your own experiences. What is the terminal velocity of a raindrop? What would its velocity be if it fell from 2000 m without air resistance?
    e. Extensions:
        i. Scaling question: why falling is so bad for a horse and really nothing at all for a mouse? To answer this question, model an animal as a cylinder with radius=1/3 length. Find the terminal velocity as a function of radius. Does your formula show that the velocity increases with animal size? How would you explain why it is so bad for horses to fall and not mice?
        ii. Terminal velocity can also occur when something moves upward. If you hold an air-filled beach ball far underwater and then let it go, it will rise to the top of the water. Find the terminal velocity for such a beach ball. You need to generalize terminal velocity to be the velocity when acceleration is zero – but here you have more than just two forces, so all forces need to be taken into account.
        iii. Impact: (This requires knowledge of the impulse-momentum theorem.) When the animal hits the ground, the ground exerts a force (and therefore a pressure on the animal). Assuming that the impact lasts .01 seconds, what force must be exerted on the cat? What is the pressure if the cat is spread eagle? What is the pressure if the cat hits only with its four paws?

2. **Hydrodensiometry.** Big question: How can we obtain a good estimate of percentage of body fat in a living person? What are the biggest assumptions in these calculations?
    a. Biology overview: For medical reasons or athletic training reasons, we often need to know the percentage of body fat in an individual. Reference: “Densitometric analysis of body compositions: revision of some quantitative assumptions”; Brozek, Grande, Anderson & Keys; Annals New York Academy of sciences.
    b. Data. The following table gives the percentages of different tissue types of the “reference human,” and the equation then givens the density of the reference individual. Siri WE (1961). "Body composition from fluid spaces and density: Analysis of methods". In Brozek J, Henzchel A. *Techniques for Measuring Body Composition*. Washington: National Academy of Sciences. p. 224–244.

Tissue	Percentage by mass	Density (kg/m^3)
Water	63%	1000
Fat	15%	900
Protein	16%	1340
Mineral/bone	6%	3100

    The Siri formula for percentage body fat (by mass) is given as

$$\% \text{ fat by mass} = \frac{495 \text{ g/ml}}{\rho} - 450$$

    where ρ is the density of the person in g/ml. This formula is derived using the assumption that ρ_{fat}=.9 g/ml and ρ_{lean}=1.1 g/ml.

    c. Physics overview: If an object is in a fluid, the fluid will exert a buoyant force on it, opposing the weight. If you place the object on a scale, the scale will read the normal force exerted by the scale required to keep the object at rest. Draw a free body diagram of a person on a scale, totally submerged in water.
    d. Question: If the scale reads (when the person is totally submerged) = .05*weight
        i. What is the density of the person?
        ii. What is their percentage body fat?
    e. Extensions
        i. Conceptual: When these measurements are made, the patient must get as much air out of their lungs as possible. If a person has air in their lungs (men typically have 4.8 liters, women 3.1 liters), how will that affect this measurement? Will they seem to have more or less fat than they actually do?
        ii. Conceptual: From your own knowledge of different body types, think of 2-3 different body types for whom the Siri formula will be in significant error?

iii. If you find from hydrodensitometry that a person's density is 1130 kg/m^3, what does the Siri formula give you as their % body fat? How do you make sense of your answer?

iv. Really challenging: Derive the Siri formula starting with the definition of density, and replacing volume with mass/density.

v. Plethysmography is a completely different method for measuring the volume of a person, which will in turn give the density=mass/volume, and then the Siri formula can be used. This is better for many people, since it does not require being submerged, but simply sitting in a "bod pod", which is like a small space capsule. Assuming the volume can be measured accurately, does this method for calculating body fat also have built in assumptions?

vi. Challenging: Plethysmography works by putting a person in a chamber with two compartments. The size of the compartments can be changed using a diaphragm, but no air leaks out. Using the ideal gas law, and knowing that pressures in both compartments can be measured, how can we get the volumes from the pressure? You have to take more than one pressure measurment.

vii. There is one final method to measure body fat, using alternating current. Find an article about this method.

3. **Analytic UltraCentrifugation.** Big question: How do centrifuges work? What can they tell us? Why do centrifuges sometimes have to spin so fast (sometimes 10,000's of revolutions per minute) so long (for hours) to do the job?
   a. Biology overview: Analytical Ultracentrifugation (AUC) is a powerful lab technique that allows researchers to characterize macromolecules. A solution is put in a centrifuge (check the internet for videos) and spun at high rotation rates; as the centrifuge spins, the larger molecules move toward the edge faster than the smaller ones, and this allows separation of molecules of different sizes. The molecules quickly reach terminal velocity, and the velocity can be measured using a microscope. The terminal velocity of the particle characterizes its mass. This method is quite flexible in that it works with many kinds solutions and many sizes of macromolecules. Also, AUC can be performed with "biologically relevant solution conditions" and is non-destructive so the sample can be reused in other tests.

      Very large molecules can be separated without a centrifuge, just by letting them fall in the liquid under the force of gravity. This system is simpler than the rotating system and will be treated first.

      AUC has uses outside of biology as well. For example, it is essential to characterize the size of small molecules in the manufacture of paint, ink, abrasives, etc. to make sure that the product functions as intended. (reference: Prof. Tom Laue, UNH).

b. Data: Possible solutions are fresh water (density = 1 g/cc, viscosity = 10^{-3}Pa *s), salt water (density = 1.02 g/cc, viscosity =1.05 *10^{-3}Pa *s). Molecules under consideration: hemoglobin (density=1.4 g/cc, radius = 2 * 10^{-9}m), silica (density =1.3 g/cc, radius = 10^{-6}m).

c. Physics overview:

   i. Drag force and Reynolds number: Objects moving in fluids are subject to both pressure drag (which increases as v^2) and viscous drag (which increases as v). Reynolds number is an approximate ratio of these two forces, and helps you decide which of these should be considered in your calculation (sometimes both are important).

   ii. Terminal velocity: This occurs anytime we have a drag force that depends on velocity and the net force on the object is zero.

   iii. Moving in a circle: In order for an object to move in a circle at a constant speed, it needs and acceleration = ω^2r.

   iv. Pressure in a rotating fluid: You are already familiar with pressure in a static fluid, where the pressure increases with depth in order to balance the weight of the fluid above; this then gives rise to a buoyant force. There is also a pressure difference for fluids moving in a circle in order to provide the net force to move the fluid in a circle. The resulting buoyant force is $\rho_{\text{fluid}} V_{\text{fluid displaced}} \omega^2 r$, where ω is the rotation speed in radians/second, and r is the radius of object in the centrifuge (this is exact only when the object's size is much smaller than its distance from the center)

d. This first problem is about silica falling in fresh water due to the force of gravity. (We will assume that all molecules are spheres, and so you can use Stoke's formula for viscous drag.)

   i. What are the forces acting on the silica? Do we need to consider both pressure and viscous drag? Justify your answer. Draw an interaction diagram and free body diagram.

   ii. What is the terminal velocity for the silica?

   iii. How long will it take for silica that started at the top to fall 0.30m? Assume that terminal velocity is reached immediately.

   iv. Now consider hemoglobin. What are the forces acting on the hemoglobin? Do we need to consider both pressure and viscous drag? Justify your answer.

   v. What is terminal velocity for hemoglobin? How long will it take for hemoglobin that started at the top to fall 0.30m? Assume that terminal velocity is reached immediately.

   vi. The calculation that we have done assumes we know the radius of the molecule. Typically we know an approximate radius, and want to find a more exact value. Outline how you would go from knowing the density and terminal velocity of the molecule, the density and the viscosity of fluid to finding the radius of the molecule.

        vii. Looking back at your data, is this a practical method for finding the size of silica? Of hemoglobin? Explain your reasoning

e. Now we consider the same ideas, but in a centrifuge. Ignore gravity unless otherwise stated. Assume we have hemoglobin in salt water.
    i. What are the forces acting on the hemoglobin? Do we need to consider both pressure and viscous drag? Justify your answer. Draw an interaction diagram and free body diagram.
    ii. If we want the hemoglobin to move 1 cm in 2 hours after terminal velocity is reached, what must be the rotation rate of the centrifuge? Take the size of the centrifuge to be 4 cm in radius.
    iii. We ignored gravity so far. How far would the hemoglobin fall vertically in this same 2 hours? (Use your answers to the last question). Are we safe ignoring gravity in this case? Explain.

f. Extensions
    i. Life at low Reynolds number is nothing like what we have experienced. How do these calculations help you understand these differences better, if they do?
    ii. Are there any particle sizes for which neither centrifugation nor falling under gravity would work? Hint: for what size molecules do you get comparable terminal velocities in both cases?
    iii. Advanced: In the literature there is a quantity called the sedimentation coefficient that is defined as $s=v_{terminal}$/acceleration (due to either gravity or spinning). It has units of time, and the standard unit of 1 Svedberg=10^{-13}s.
        1. What is the meaning of this time? Is it the time to fall? To reach terminal velocity? Something else?
        2. From your equation for terminal velocity, find the value of *s* in terms of other constants in this problem.
        3. What is the value of knowing the sedimentation coefficient for a particular system? (Calculate *s* for the situations given above – are they different?)
    iv. Biologists sometimes refer to both sedimentation and AUC as sedimentation and both forces as gravity. In what way are they similar? Different? There have been plans to simulate gravity in space with a rotating space ship. If the ship has a 15m radius, how fast would it have to rotate to simulate gravity on earth?

4. Fish Bladders: still to come…